

## Theoretical noise sensitivity of BRDF and albedo retrieval from the EOS-MODIS and MISR sensors with respect to angular sampling

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**Abstract.** The sensitivity of the semiempirical RossThick-LiSparse Ambrals BRDF model to random noise in observed multiangular reflectances was investigated through a study of the impact of angular sampling. The mathematical properties of (linear, additive) kernel-driven BRDF models allow the analytical derivation of so-called weights of determination or noise amplification factors which quantify the uncertainty in retrieved parameters such as nadir-view reflectance or albedo at various solar zenith angles, or in the BRDF model parameters themselves. The study was carried out using simulated angular sampling for the MODIS and MISR instruments to be flown on NASA's Earth Observing System AM-1 platform, as a function of latitude, day of year and sampling period. A similar study was carried out for comparison using the modified RPV BRDF model, a multiplicative model. Results show that the retrieved parameters, reflectance and albedo can be expected to have noise amplification factors that are less than unity, indicating that the retrievals are stable with respect to random noise under the angular sampling schemes occurring. The BRDF model parameters themselves were found to be more susceptible to noise than many of the derived products, especially for the modified RPV model. The effect of different angular sampling regimes on the uncertainty of derived information was further explored. This study provides an indication of the reliability to be expected from the operational BRDF/albedo products from the MODIS and MISR instruments. The findings may qualitatively also apply to AVHRR, SPOT VEGETATION and similar satellite angular sampling regimes.

### 1. Objectives

Advanced algorithms for the radiometric characterization of the Earth's land surface are currently being implemented in the data processing streams of a new generation of remote sensing instruments. Earth surface reflectance is specified as the bidirectional reflectance distribution function (BRDF) and its angular and

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spectral integrals, from which land surface albedo can be estimated (Lewis and Barnsley 1994). The BRDF is derived by inverting observed multiangular reflectances to derive the parameters of a semiempirical BRDF model. These parameters provide a spatial, temporal, spectral and directional quantification of land surface reflectance for use in ecosystem and atmospheric simulation and prediction models.

One key issue in BRDF/albedo data processing is being able to provide an estimate of the accuracy of the retrieved values. To validate a global BRDF/albedo product with ground-based measurements is notoriously difficult due to questions of spatial scaling, land surface inhomogeneity, the expense of instrumentation and the difficulty of obtaining reliable multiangular datasets. A solid theoretical understanding of error sources in the derivation of the products and quantitative error analyses through numerical simulation are therefore indispensable for deciding upon an algorithm and for attaching quality measures to the product generated.

This paper is the third in a series that studies systematically issues of BRDF and albedo retrieval accuracy with respect particularly to angular sampling (Hu *et al.* 1997, Lucht 1998). This research is aimed at understanding the controlling influences on operational BRDF/albedo processing, and to characterize the expected product accuracies. In this context, the current paper investigates the influence of the angular sampling regime and random uncertainties in observed multiangular reflectances on the quality of BRDF model parameters, and on derived reflectances and albedo.

Noise sensitivities were derived using realistic angular sampling distributions for two new remote sensing instruments co-located on NASA's Earth Observing System (EOS) AM-1 platform, to be launched in late-1999: the Moderate Resolution Imaging Spectroradiometer (MODIS) (Justice *et al.* 1998) and the Multiangle Imaging Spectroradiometer (MISR) (Diner *et al.* 1991). The semiempirical RossThick-LiSparse Ambrals BRDF model (Wanner *et al.* 1995), which is the basis for operational production of a MODIS BRDF/albedo product (Wanner *et al.* 1997), is mainly investigated. However, for comparison results are also generated for the modified Rahman-Pinty-Verstraete (RPV) BRDF model (Rahman *et al.* 1993, Engelsen *et al.* 1996), which is the model used in generating a BRDF/albedo product for the MISR instrument.

The results obtained are also relevant for the Advanced Very High Resolution Radiometer (AVHRR), the SPOT VEGETATION instrument, and the Medium Resolution Imaging Spectrometer (MERIS), which have angular sampling patterns similar to that from MODIS due to their across-track view angle variations. The results found may therefore also give a good indication of what should be possible with these instruments, which, for AVHRR at least, is of great importance due to the length of the existing data record from this instrument.

The following section provides a discussion of the types of error and uncertainty that must be considered and places the work in the context of EOS. Section 3 outlines the method and presents results for the Ambrals model. Section 4 presents and discusses results for the modified RPV model. Section 5 provides a summary and draws conclusions from this work.

## 2. Context

### 2.1. Residual error in BRDF/albedo retrieval in relation to angular sampling

When inverting multiangular reflectances for the parameters of a BRDF model, a residual error remains which indicates deviations of the model fit from the observed reflectances. Assuming unbiased observations, such an error may have two principal

causes, namely a partial inability of the model to fit the observed BRDF (i.e. a wholly or partially inappropriate model), or error (uncertainty or random noise) in the observed reflectances. In the first case, the residual error is systematically distributed in angular space, in the second, randomly. Note that for sparse sampling it may sometimes be difficult to discriminate between the two cases. Where dense angular sampling is available, the former should be able to be distinguished by a slowly decreasing residual autocorrelation, indicating sectors of consistent over- and under-estimation, rather than random noise.

In either of these cases, the most important question that arises is how the error affects the quality of the retrieved model parameters and predicted reflectances as one would like to know the accuracy of reflectances and derived albedo especially when the model is used to predict reflectances (extrapolate observations) away from the angles where observations were made.

It is clear that BRDF/albedo retrieval accuracies, and hence the impact of noise, depend on the angular distribution of the available observations. Even in the *absence* of model inadequacies and noise, problems may occur when the BRDF model parameters are insensitive to the observations in particular areas of angle space and the angular sampling available is limited to those areas (for example, all samples are away from the solar principal plane). In that case, the observations do not provide the information required to reliably distinguish different possible values of the parameters. In the presence of partial, if small, model inadequacies and of noise in the data, and under conditions of sparse angular sampling as are typical for remote sensing, such problems may be enhanced.

The basic ability of the Ambrals and the modified RPV BRDF model to fit naturally occurring BRDF shapes was studied by Hu *et al.* (1997), Rahman *et al.* (1993) and Engelsen *et al.* (1996) and was found to be generally adequate. Inversion root mean square errors for a number of field-measured multiangular reflectance datasets representing different land-cover types were small.

BRDF and albedo retrieval accuracies under sparse angular sampling were systematically investigated by Lucht (1998) using synthetic noise-free BRDFs and a great variety of realistic MODIS and MISR angular sampling distributions, and by Privette *et al.* (1997) using field-observed BRDFs and sparse generic angular distributions. Both concluded that the RossThick-LiSparse Ambrals and the RPV BRDF models are capable of modelling reflectance and albedo both at observed and at extrapolated viewing and illumination angles even under sparse angular sampling. Errors were found to be within 10% relative in a majority of cases.

The current investigation adds to these studies an assessment of the effect of random noise variations in the data and the effects of different angular sampling patterns encountered at different latitudes and times of year upon the accuracy of the retrieved BRDF model parameters and derived reflectances and albedo.

In concluding this brief overview, it should be stressed that these are accuracy issues related only to angular sampling. A different form of inaccuracy may be introduced into a BRDF/albedo product by a systematic bias in the observations, for example due to systematic errors in atmospheric correction that depend on the zenith angle. But such errors are not of interest here. For example, Hu *et al.* (1999a) investigated the error introduced into Ambrals model BRDF retrievals by atmospheric correction procedures that assume a Lambertian surface. Errors due to uncertainties in the aerosol optical depth were studied for the modified RPV model by Diner *et al.* (1996).

## 2.2. *Random noise effects in BRDF/albedo retrieval*

BRDF/albedo inversions will only be reliable if they display a certain robustness to uncertainty, or error, in each individual observed reflectance. Such errors will be considered random if their cause is not dependent on the angle of observation and they affect all observations in the same way.

A random noise component in the observations may be caused by the simultaneous presence of several effects impacting on different observations to various degrees. In a multivariate dataset of multiangular reflectances, such as will be the basis of MODIS, AVHRR or SPOT VEGETATION BRDF inversions, geolocation inaccuracies will introduce an error that potentially is different for each orbit. Cloud-edge scattering into the field of view and sub-resolution residual clouds that were not removed by the cloud-screening performed lead to increases in some reflectances. The corresponding undetected sub-resolution cloud shadows lead to decreases in other reflectances. In multivariate multiangular databases, varying atmospheric conditions from one day to another may lead to slightly different accuracies in the atmospheric correction scheme employed on different observations. Random noise effects from varying atmospheric conditions are expected in data that were only bulk corrected, for example because no concurrent atmospheric data were available (as is the case with much of the historical AVHRR processing). There may be day-to-day changes in surface condition as well, due for example to the difference in soil brightness between wet and dry days, differences in vegetation canopy properties between hot and cool days, and effects related to slight phenological changes during the compositing period. Where such influences lead to outlier data, they may be detected and the corresponding data eliminated from analysis. The remaining data may generally be approximated as normally distributed random noise, assuming the model used is appropriate to the description of the BRDF.

Generally, fitting a BRDF model to multiangular observations may be regarded as a special type of intelligent compositing, where effects on the individual data are averaged and an average reflectance is produced. Such a compositing may actually compare favourably with the traditional maximum-value compositing (Hu *et al.* 1999b). However, the question remains, what is influence of a noise-like component in the observed reflectances on retrieved BRDF and albedo parameters? The effect expected varies as a function of the angular distribution of samples and is therefore different for each practical application. Both the density and the distribution of samples are relevant.

## 2.3. *The EOS context*

The MODIS sensor is an across-track imager with a swath width of 2330 km and almost daily global coverage. A multiangular dataset of reflectance observations is assembled over a period of time, for example 16-days (the orbital two-repeat cycle). MISR is an along-track imager with a swath width of 364 km using four fore-, four aft- and one nadir-pointing camera. A set of nine multiangular observations is acquired instantaneously, but due to the narrow swath width it requires 9 days to cover the globe. The polar orbiting EOS-AM-1 platform carrying MODIS and MISR will have a 10:30 morning equatorial crossing time. Combined over a period of time, these two instruments provide strings of multiangular observations across the viewing hemisphere that are almost perpendicular to each other (see Wanner *et al.* (1997) for an example). Their respective azimuthal angle from the principal

plane varies with latitude and time of year, as does the mean solar zenith of the observations and the number of observations from MODIS (Barnsley *et al.* 1994).

The MODIS BRDF/albedo product will be generated from combined MODIS and MISR observations acquired over a 16-day period (Strahler *et al.* 1996, Wanner *et al.* 1997). Semiempirical RossThick-LiSparse Ambrals BRDF model parameters will be provided in seven spectral bands between 400 and 2100 nm, and in the visible, near-infrared and total shortwave broadbands. Spatial resolution of the product will be 1 km and it will be derived for the global land surface. Product prototyping has been carried out with the AVHRR (Lewis and de Lope 1997, d'Entremont *et al.* 1999, Hu *et al.* 1999b).

In the first period after the launch of MODIS, the product will be derived from MODIS observations only until a MISR data stream is established into MODIS data processing. Therefore we investigate the noise sensitivity of retrievals to angular sampling for each instrument separately as well as for the combination. This is also important because three of the seven MODIS bands have no equivalent on MISR, which has only four bands. MISR alone will therefore not be able to derive a total shortwave albedo, and the respective MODIS bands will not be able to rely on angular samples from MISR. In late 2000, a second MODIS will be launched on the EOS-PM-1 platform, which will have an afternoon equatorial crossing time of 1:30. We study whether combining MODIS-AM and MODIS-PM data is equivalent in terms of the robustness of the angular sampling provided to noise sensitivity to combining MODIS-AM and MISR. This would give two instruments with different angular sampling azimuths but identical spectral bands. Finally, we investigate combining all three instruments to make full use of all the moderate resolution EOS sensors that will be available in the near future.

### 3. Noise sensitivity of the RossThick-LiSparse Ambrals BRDF model

#### 3.1. Model

The Ambrals BRDF model is a semiempirical kernel-driven (or linear) BRDF model. This type of model was first suggested by Roujean *et al.* (1992) for the rapid inversion of multiangular satellite observations over potentially homogeneous vegetated terrain. The reflectance  $R$  is modelled as a linear superposition,  $R = \sum f_i k_i$ , of mathematical functions  $k_i$  (kernels) representing basic but physically distinct (but non-orthogonal) BRDF shapes as a function of viewing and illumination geometry. In model inversion, the weights  $f_i$  are the model parameters retrieved.

BRDF models of this class possess several useful advantages (Wanner *et al.* 1995). For example, they may be inverted analytically, which makes inversions robust and fast. Albedo is parametrized by the same weights as the BRDF, which leads to an identity of the BRDF with the albedo model. The kernel functions, which are nonlinear functions of the viewing and illumination geometry only, may be pre-computed, and the full scope of linear systems theory, which is very well developed, may be brought to apply.

The Ambrals RossThick-LiSparse BRDF model consists of three kernels (Wanner *et al.* 1995). The first is a constant, unity, representing isotropic scattering. The second is the RossThick kernel, which was derived by Roujean *et al.* (1992) from a single scattering approximation to radiative transfer theory for homogeneous layered plant canopies by Ross (1981). This kernel may be taken to describe the volume scattering contribution in a scene. The third kernel is called the LiSparse kernel and was derived by Wanner *et al.* (1995) from a geometric-optical mutual shadowing

model for canopies of randomly distributed spheroidal protrusions by Li and Strahler (1992). This kernel describes surface or geometric scattering in a scene, mainly the casting of shadows by the three-dimensional crowns. Slightly restated, the LiSparse kernel quantifies light scattering effects due to inter-crown gaps, and the RossThick kernel scattering due to intra-crown gaps, i.e. gaps between the leaves. The model parameters  $f_{geo}$  and  $f_{vol}$  associated with these kernels give the respective strength of each type of scattering in the scene viewed.

### 3.2. Angular sampling, reflectances and albedo

MODIS and MISR angular sampling distributions were simulated using the Xsatview software (Barnsley *et al.* 1994). Viewing and illumination geometries were constructed for nine geographic latitudes between 80°S and 80°N, and for eight different 16-day time periods throughout the year.

Retrievals investigated are for model parameters, nadir-view reflectance, black-sky and white-sky albedo. The latter are the directional-hemispherical and bihemispherical integrals of the BRDF, representing land surface albedo in the two limiting cases of absence of diffuse illumination and of perfectly diffuse illumination. Lewis and Barnsley (1994) suggest that a weighting of these two terms by the proportion of diffuse illumination provides an accurate estimation of albedo except for relatively high solar zenith angles. The albedo for a very clear atmosphere will be an interpolated value that is close to the value of the black-sky albedo, whereas conditions under clouds will be represented by an interpolated value close to the white-sky albedo. Quantities that depend on solar zenith angle, i.e. nadir-view reflectance and black-sky albedo, are derived both at the mean solar zenith angle of observation (interpolation of observations) and for an arbitrary fixed nadir solar zenith angle (extrapolation, where the amount of extrapolation performed depends on the sun zenith angle of observation, i.e. on the latitude and time of year).

### 3.3. Method

Due to their linear mathematical form, the behaviour of kernel-driven linear models under the conditions of limited and varying angular sampling can be studied analytically. The theory of least squares and related statistical analysis permits the derivation of unbiased estimates of model parameters and linear combinations of model parameters such as reflectances at given angles and albedos. These techniques also directly provide estimates of the variance in these quantities.

Strictly speaking, the method of least squares assumes that any deviation from a perfect fit in an over-constrained case (number of samples larger than number of model parameters) is due to uncertainty (noise) in the magnitude of the observation. Related statistical theory tends to assume further that the variation in reflectance at each observation angle is normally distributed and of equal variance over the reflectance function. If the variance of the reflectance noise varies in some predictable way over the observation angles, this can be taken into account by weighting the error function. The error that is minimized in fitting the model, for example the root mean squared error (RMSE), provides an estimate of this variance in observation.

Given such noise-like effects in the observed data, the noise sensitivity of a term  $u$  (a linear model parameter or a linear combination of model parameters such as reflectance or albedo) under a particular angular sampling may be studied by computing the so-called weights of determination. Using theory that originates with Gauss (Whittaker and Robinson 1960), the uncertainty (error)  $\varepsilon_u$  expected in  $u$  may

be expressed as

$$\varepsilon_u = e \sqrt{\frac{1}{w_u}} \quad (1)$$

where  $e$  is an estimate of standard error in the observed data, and  $1/w_u$  is the weight of determination of term  $u$ . The latter is given by

$$\frac{1}{w_u} = [U]^T [M^{-1}] [U] \quad (2)$$

where  $U$  is a vector composed of the weighting of the kernels in some linear combination of the kernels resulting in the term  $u$  under consideration, and  $M^{-1}$  is the inverse matrix providing the solution of the least-squares inversion problem for the linear model. Note that the form of analysis is independent of any specific BRDF shape, in that  $M^{-1}$  is a function only of variances of and covariances between the kernels used. This is a unique feature of linear models which means that the impact of angular sampling can be studied independently of the model values. In a nonlinear model, an equivalent analysis can be undertaken through linearization, for instance using a Taylor expansion, for a particular set of model parameter values, but the impact of the angular sampling scheme will vary also as a function of the parameter values.

For linear models, to obtain, for example, the weight of determination of the parameter  $f_0 = f_{is_0}$  of a kernel-driven model,  $[U]^T = (1, 0, 0)$ . The weight of determination of directional-hemispherical reflectance at solar zenith angle  $\theta_s$  is formed from  $[U]^T = (1, \overline{k_1(\theta_s)}, \overline{k_2(\theta_s)})$ , where  $\overline{k_i(\theta_s)}$  are the respective directional-hemispherical integrals of the kernels used (which are functions of the solar zenith angle). The weight of determination of bihemispherical reflectance is formed from  $[U]^T = (1, \overline{\overline{k_1}}, \overline{\overline{k_2}})$ , where two bars stand for the respective bihemispherical integral. The weight of determination of the reflectance at some combination of viewing and illumination angles,  $(\theta_v, \theta_s, \phi)$ , is given by forming  $[U]^T = (1, k_1(\theta_v, \theta_s, \phi), k_2(\theta_v, \theta_s, \phi))$ .

The weight of determination depends on the angular sampling scheme available because  $M^{-1}$  depends on it. The weight of determination also depends on the number of samples,  $N$ , and contains the factor  $1/\sqrt{N}$ . Increasing  $N$  decreases the expected error because the errors are assumed to be randomly distributed at each observation angle. Thus, we can already begin to understand that factors such as cloud cover, which will reduce  $N$  from the maximum ideal number considered in this study to  $N'$ , will tend to increase the expected error even if the angular distribution of samples remains roughly the same. The increase in each term under consideration is given by  $\sqrt{N/N'}$ .

Given the standard deviation of the uncertainty or error in reflectance observations, the weight of determination associated with a term  $u$  of interest directly gives, in form of a factor, the standard deviation of uncertainty or error in that term. We therefore will also call the weight of determination the noise amplification factor.

The noise sensitivity of reflectance and albedo can be studied through the noise amplification factor without having to specify the magnitude of  $e$ . In practical applications, one may assume that the variance in the observations will certainly not be larger than the RMSE of the inversion indicates. This allows us to approximate  $e$  by the RMSE, although the latter may also be partly due to non-random deviations

of the model from the observations (partial inappropriateness of the model used). The estimate of uncertainty, therefore, becomes the product of RMSE and the square root of the weight of determination. Uncertainty in the model parameters tends to be reduced under sampling schemes for which the variance of each of the kernels is maximized and the covariance between the kernels minimized.

### 3.4. Results

We studied the sensitivity to random noise of the RossThick-LiSparse Ambrals BRDF model using sampling for a variety of combinations of the MODIS and MISR sensors, and for different periods of data aggregation.

Table 1 reports median values and ranges of the weights of determination found both for interpolated and extrapolated nadir-view reflectance and albedo, and for the model parameters. Note that the parameter  $f_{iso}$  is identical to nadir-view nadir-sun reflectance due to the normalization of the RossThick-LiSparse model and, hence, is not listed separately. The median, characterizing the typical case, is taken with respect to sampling for the different latitudes and times of year investigated. The range refers to the values enclosing two-thirds of the data for all latitudes and times of year. The table reports results for six different angular sampling scenarios involving MODIS-AM, MODIS-PM and MISR. The first column reports the 16-day MODIS-AM/MISR combination envisaged for the full MODIS BRDF/albedo product. The second reports 9-day MISR-only sampling, which corresponds to one MISR observation for each global location and is the scenario for the MISR BRDF/albedo product (which, however, uses the modified RPV BRDF model). The 16-day MODIS-AM-only angular sampling reflects the condition of the MODIS product without a MISR data stream. These results will be similar to those obtained from AVHRR or VEGETATION sampling over this time period. MISR-only 16-day

Table 1. Noise amplification factors for the RossThick-LiSparse Ambrals BRDF model.

		MODIS-AM/ MISR				MODIS- AM/PM	MODIS-AM/ PM/MISR
		16-day	MISR 9-day	MODIS-AM 16-day	MISR 16-day	16-day	16-day
<b>Medians</b>							
Interpolation	Rnad	0.23	0.30	0.40	0.31	0.23	0.16
$\theta_s = \langle \theta_s \rangle$	bsa	0.17	0.23	0.35	0.21	0.20	0.12
Extrapolation	Rnad	0.46	0.63	1.17	0.58	0.67	0.36
$\theta_s = 0$	bsa	0.21	0.28	0.33	0.29	0.19	0.15
Global, $\int \theta_s, d\theta_s$	wsa	0.34	0.42	0.99	0.34	0.55	0.27
Parameters	$f_{vol}$	0.89	1.25	2.01	0.97	1.19	0.73
	$f_{geo}$	0.27	0.37	0.68	0.28	0.39	0.22
<b>Ranges</b>							
Interpolation	Rnad	0.18–0.28	0.25–0.36	0.29–0.44	0.25–0.38	0.17–0.25	0.12–0.18
$\theta_s = \langle \theta_s \rangle$	bsa	0.13–0.18	0.18–0.25	0.30–0.37	0.17–0.22	0.18–0.21	0.10–0.13
Extrapolation	Rnad	0.40–0.55	0.51–0.83	0.69–2.48	0.47–0.70	0.39–1.39	0.27–0.51
$\theta_s = 0$	bsa	0.17–0.36	0.24–0.50	0.28–1.29	0.23–0.45	0.16–0.72	0.12–0.28
Global, $\int \theta_s, d\theta_s$	wsa	0.19–0.58	0.25–0.80	0.51–1.10	0.24–0.79	0.30–0.65	0.14–0.43
Parameters	$f_{vol}$	0.33–1.76	0.48–2.48	1.21–3.52	0.37–3.28	0.72–1.97	0.28–1.08
	$f_{geo}$	0.20–0.31	0.27–0.42	0.57–1.07	0.21–0.39	0.34–0.62	0.17–0.24

Rnad, reflectance at nadir view angle; bsa, black-sky albedo; wsa, white-sky albedo;  $f_{vol}$ , volume scattering kernel coefficient;  $f_{geo}$ , surface scattering kernel coefficient.

sampling is given for comparison. The last two columns investigate whether MODIS-PM will be an equivalent substitute for MISR, and whether using all three instruments together will provide a significant decrease in noise sensitivity due to improved angular sampling.

The noise amplification factors listed in table 1 are almost all smaller than 1. This indicates stability of the reflectances, albedos and parameters investigated with respect to random noise effects in the observations for the expected angular sampling regimes. The magnitude of the noise-induced variance in these derived quantities will be less than that in the original reflectances; i.e. the retrievals are not rendered meaningless by noisy inputs. This is a very positive finding for the retrievals intended from MODIS and MISR.

It is interesting to note that the model parameters are notably more noisy than the physical quantities, reflectance and albedo, derived from them. This indicates a partial insensitivity of reflectance and albedo to the values of the model parameters. Parameters are seen to be slightly redundant in the sense that they may trade off magnitude between each other without severely affecting BRDF shape. In applications, one should, therefore, plot linear combinations of the parameters rather than the parameters themselves. One may note that the black-sky albedo at the mean solar zenith angle tends to be the most stable product derived.

While noise sensitivities for albedo are generally low throughout, nadir-view reflectance is more susceptible to noise when extrapolated to nadir solar zenith angle than at the solar zenith angle of observation. Noise amplification factors are still less than 1, but this may be a factor in deciding whether to perform BRDF corrections for view angle effects only or for solar angle as well.

The 16-day angular sampling corresponding to the MODIS BRDF/albedo product produces noise amplification factors that are smaller than unity for all quantities studied. There is a moderate improvement of the stability of retrievals with respect to using only MISR data, but the differences are not large. On the other hand, using only MODIS data for the same time period leads to still acceptable, but noticeably larger weights of determination. This clearly shows that with respect to retrieval stability, MISR angular sampling is superior to MODIS angular sampling as it is more often closer in azimuth to the solar principal plane (Barnsley *et al.* 1994). Adding MISR data to the MODIS data stream will greatly improve the quality of the products derived. Using MISR alone is, however, not a sufficient substitute for using MODIS as well because MODIS, as a 36-channel instrument, will be able to characterize the atmosphere more precisely than MISR, has three additional dedicated land bands in the near-infrared which MISR does not feature, allowing estimation of shortwave broadband albedo.

However, it is clear from table 1 that the susceptibility of the MODIS-only bands to noise-like effects in the data is larger than those shared by MODIS and MISR. Still, black-sky albedo at extrapolated and interpolated solar angles, and nadir-reflectance at the solar zenith angle of observation may be retrieved with confidence from MODIS-only sampling. Extrapolated nadir-view reflectance and white-sky albedo are more susceptible to noise, but not more so in the median than the observed reflectances themselves.

The volume scattering model parameter is much more sensitive to noise than the geometric scattering parameter. This is mainly due to the fact that discrimination of volume scattering is best at large solar zenith angles, where sampling is sparse, whereas shadowing effects are more evident at moderate angles.

Using MODIS-PM instead of MISR improves the noise stability of the retrievals considerably, although not to the full extent that using MISR does, the advantage of MODIS-PM being, however, that it will feature the same seven land-designated spectral bands as the MODIS-AM instrument, whereas MISR has only four bands. Using all three instruments is clearly best, owing to the large number of samples and improved angular sampling scheme available from this combination.

The ranges of values given indicates that under some sampling conditions noise sensitivity may be an effect to be reckoned with in a BRDF/albedo product. Noise amplification factors larger than unity occur mainly when sampling does not include the principal plane for either MISR or MODIS, and when the solar zenith angle is very large.

Figure 1 depicts the dependence of the noise amplification factors summarized in table 1 on latitude and time of year. Curves represent different days in the first half of the year. Noise sensitivity depends on latitude and time of year since these determine the solar zenith angle for interpolation and the distance to be extrapolated from the solar zenith angle to nadir sun for extrapolation. The relative azimuth of the observations also depends on these variables. We can also note that the number of samples available increases with latitude due to convergence of the orbital tracks (Barnsley *et al.* 1994). Panel (*f*) shows the error expected when extrapolating black-sky albedo in sun zenith angle for different latitudes and sampling in the first 16-day period of the year. It demonstrates that extrapolation towards nadir is less problematic than extrapolation to large zenith angles for all latitudes, the beginning of the rise being determined by the sun zenith angle at which the observations were made.

Figure 2 shows in the top two panels the noise sensitivity of the model parameters themselves (note that the isotropic parameter is identical with nadir-view, nadir-sun reflectance, shown in figure 1(*c*)). For the reasons discussed above, they are more susceptible to noise than reflectance and albedo, particularly for low solar zenith angles. The reason is that the shape of the BRDF is more pronounced at large solar angles, allowing better discrimination of parameters (higher variance of kernels). The corresponding nadir reflectance, however, is not overly sensitive to the exact value of the parameters, which mainly influence off-nadir reflectance.

### 3.5. *The influence of clouds*

All numbers given refer to angular sampling assuming no loss of observations to clouds. Such numbers are important because they are the theoretical baseline accuracies to be taken into consideration even under ideal conditions. Realistically, however, the true noise sensitivities will be larger due to clouds. Depending on the latitude and the season, between one-third and two-thirds of the observations are likely to be lost because of clouds.

MODIS observations will approximately be dropped at random with respect to their distribution in angular space as MODIS acquires daily observations with varying viewing geometry. MISR acquires multiangular observations instantaneously, but observes a given location only at intervals of several days. However, in a 16-day period there will be two or more viewing opportunities for MISR which will produce a very similar angular distribution.

Consequently, if half of all observations are lost to clouds, the angular distribution of sampling will be approximately the same for the MODIS/MISR sensor combination even when the number of observations drops, as long as MISR manages to acquire at least one set of observations. Only if MISR acquires no observations over

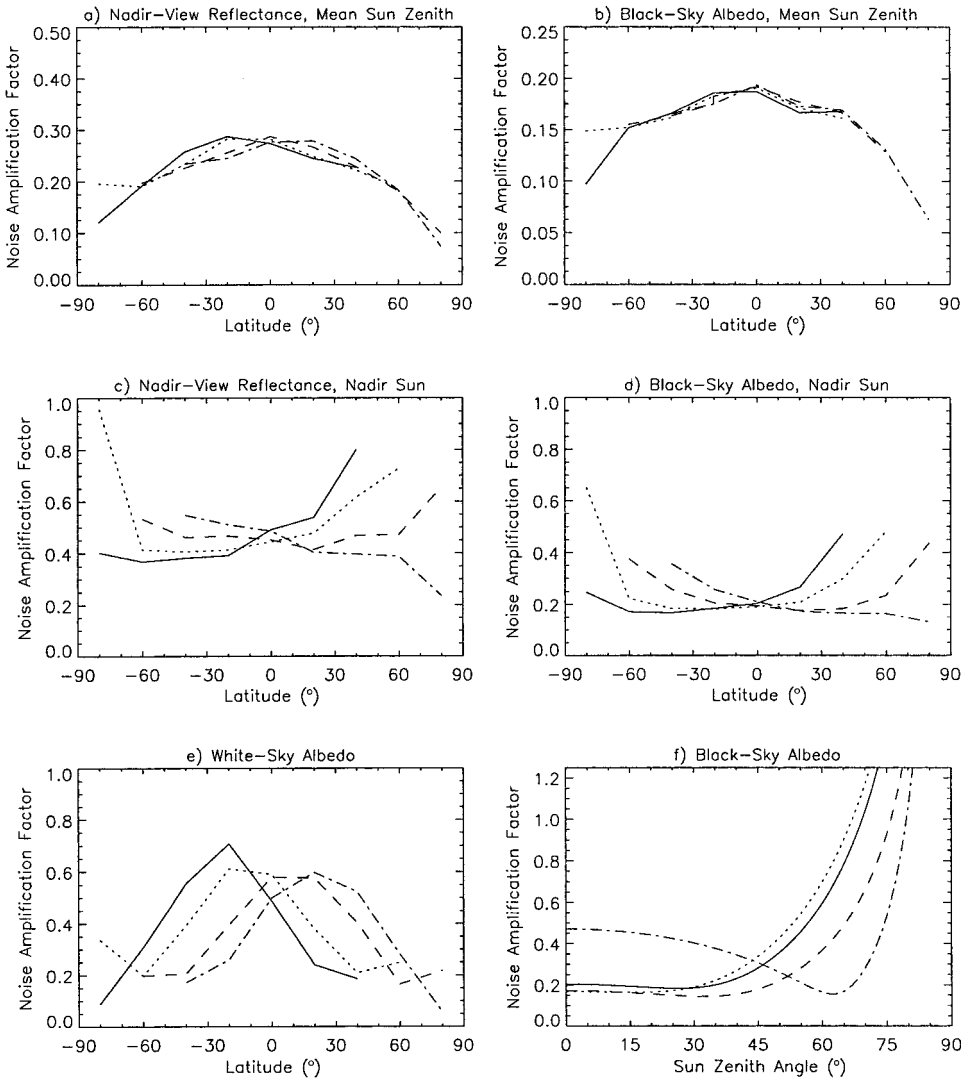


Figure 1. Noise sensitivity of the Ambrals RossThick-LiSparse BRDF model. Weights of determination (noise amplification factors) are shown as a function of latitude for different 16-day time periods throughout the first half of the year, the time progressing through solid, dotted, dashed and dashed-dotted curves (days of year 0, 48, 96 and 144). Panel (f) shows the noise sensitivity of black-sky albedo extrapolation as a function of sun zenith angle for sample latitudes  $-60$ ,  $-40$ ,  $0$  and  $40^\circ$  latitude (solid, dotted, dashed, dashed-dotted curves), for sampling during the first 16-day period of the year. The terms ‘mean sun zenith’ and ‘nadir sun’ refer to evaluations at the mean sun zenith angle at which the observations were made at each respective latitude and for the case of a solar zenith angle extrapolated to nadir.

the 16-day period will the noise sensitivity degrade to MODIS-only numbers. These will deviate from the 16-day case as  $\sqrt{N/N'}$ , where  $N$  is the maximal number of observations and  $N'$  the number actually obtained. Assuming  $N' = N/2$ , the worst case of sampling loss due to clouds is then given by the numbers in the 16-day MODIS-only column of table 1, multiplied by 1.414. Mean noise sensitivity factors

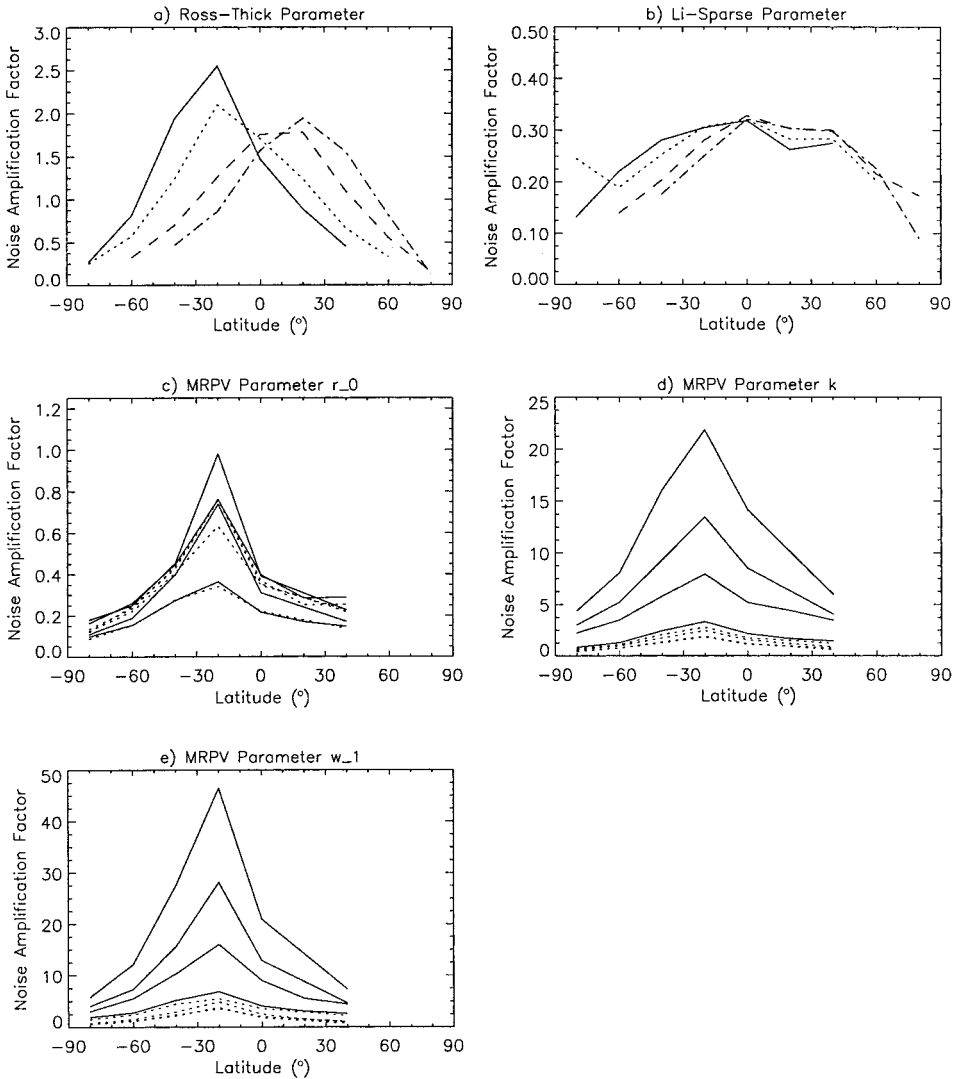


Figure 2. Noise sensitivity of the model parameters of the Ambrals RossThick-LiSparse BRDF model and the modified RPV BRDF model. Weights of determination (noise amplification factors) are shown as a function of latitude. For the Ambrals model they are shown for different 16-day time periods throughout the first half of the year (days of year 0, 48, 96 and 144 shown as solid, dotted, dashed and dashed-dotted curves), for the modified RPV model for sampling on the first of these 16-day periods and for the red (solid curves) and near-infrared band (dotted curves) of four different land-cover types (Ambrals model analysis is independent of band or land-cover type due to the mathematical properties of kernel-driven models).

for interpolated nadir-view reflectance in this worst case will be 0.44, of albedo at the mean sun angle of observation 0.30, of extrapolated nadir-view reflectance 1.66, of nadir-sun black-sky albedo 0.47, and of white-sky albedo 1.4. However, this case will not occur too frequently, as at least one MISR observation should generally be available.

Overall, the noise sensitivity of the RossThick-LiSparse Ambrals BRDF model

with respect to predicting BRDF and albedo at the sun zenith angle at which the observations were made and extrapolated to a nadir solar zenith angle is such that noise-like effects in the reflectance observations usually lead to an error in the derived quantities that is smaller than that present in the reflectances. Under conditions where observations will be lost to clouds, the sensitivity will increase, but the noise inflation factor will still be mostly smaller than unity, and only somewhat larger in the worst cases. It is not surprising that problems will occur in cases of unfavourable angular sampling distribution (large end of the range of values) under conditions of severe loss of observations to clouds.

A similar study was also conducted for other possible choices and combinations for the Ambrals model kernels, additionally utilizing the so-called RossThin and LiDense kernels (Wanner *et al.* 1995), which describe sparser leaf and denser crown canopies, respectively. The results were generally very similar to those reported here.

#### 4. Noise sensitivity of the modified RPV model

The noise sensitivity of the RPV BRDF model by Rahman *et al.* (1993) as modified by Martonchik (Engelsen *et al.* 1996) was also investigated for comparison and in order to reveal whether the properties found are similar to those of the Ambrals model. This may help answer the question to what extent the noise sensitivity behaviour found is typical of a particular model or whether it might characterize 3-parameter models in general and be rather generically a function only of angular sampling.

The RPV model is a nonlinear model that gives reflectance as a product of three terms: modelling the bowl shape of many BRDFs, an asymmetry between forward and backward scattering, and a hotspot. The modified RPV model is a close approximation to this which allows rapid inversion. After taking a logarithm of the modelled reflectance, it remains partially nonlinear, making an analytical investigation of noise sensitivity along the lines employed for the Ambrals BRDF model impossible. However, an equivalent weight of determination may be constructed from the RMSE and the variation found in the derived quantity, albedo or reflectance, using equation (1). This was done by computing 250 random realizations of noisy data for each of five magnitudes of noise up to 5% absolute of the reflectance (keeping the resulting reflectances non-negative).

Due to the nonlinear nature of the modified RPV model, the analysis also depends on land-cover type and wave band. It was carried out for the red and the near-infrared bands using four different datasets measured by Kimes (1983) and Kimes *et al.* (1985, 1986), namely BRDF observations of corn, a ploughed field, a hardwood forest and a grass lawn. These types were chosen to represent different types of naturally occurring BRDFs, giving an indication of the kind of results expected for observations of barren and vegetated surfaces.

Table 2 gives medians and ranges of the inferred equivalent weights of determination in the red and near-infrared band for 16-day MODIS-AM and MISR angular sampling. The corresponding weights of determination for the RossThick-LiSparse model are also given for comparison. The ranges given for the medians refer to the different land-cover types.

Results are generally similar for the two models, with the modified RPV noise amplification factors tending to be slightly lower. This demonstrates that both models are equally stable in BRDF/albedo retrievals with respect to random noise. However, the table also reveals that two of the three RPV model parameters are extremely susceptible to noise, which fortunately does not translate into noisy reflectances and

Table 2. Noise amplification factors for the modified Rahman-Pinty-Verstraete BRDF model.

MODIS-AM/MISR 16-day sampling		Modified RPV		RossThick-LiSparse
		Red	NIR	Red/NIR
<b>Medians</b>				
Interpolation	Rnad	0.21– 0.28	0.19–0.22	0.23
$\theta_s = \langle \theta_s \rangle$	bsa	0.08– 0.16	0.04–0.06	0.17
Extrapolation	Rnad	0.32– 0.49	0.31–0.49	0.46
$\theta_s = 0$	bsa	0.11– 0.20	0.08–0.13	0.21
Global, $\int \theta_s d\theta_s$	wsa	0.19– 0.23	0.17–0.19	0.34
<b>Parameters</b>				
	$\tau_0$	0.17– 0.29	0.16–0.25	—
	$k$	1.70– 9.86	0.89–1.41	—
	$w_1$	3.12–12.12	1.15–2.71	—
	$f_{vol}$	—	—	0.89
	$f_{geo}$	—	—	0.27
<b>Ranges</b>				
Interpolation	Rnad	0.15– 0.31	0.15–0.26	0.18–0.28
$\theta_s = \langle \theta_s \rangle$	bsa	0.07– 0.18	0.03–0.09	0.13–0.18
Extrapolation	Rnad	0.28– 0.76	0.27–0.76	0.40–0.55
$\theta_s = 0$	bsa	0.08– 0.39	0.04–0.33	0.17–0.36
Global, $\int \theta_s d\theta_s$	wsa	0.10– 0.49	0.06–0.36	0.19–0.58
<b>Parameters</b>				
	$\tau_0$	0.13– 0.51	0.12–0.47	—
	$k$	1.15–15.25	0.45–2.00	—
	$w_1$	2.20–28.20	0.59–4.02	—
	$f_{vol}$	—	—	0.33–1.76
	$f_{geo}$	—	—	0.20–0.31

Land cover types (Kimes 1983, Kimes *et al.* 1985, 1986): corn, lawn, ploughed field, hardwood forest.

Rnad, reflectance at nadir view angle; bsa, black-sky albedo; wsa, white-sky albedo;  $\tau_0$ , base reflectance coefficient;  $k$ , BRDF slope coefficient;  $w_1$ , forward/backward scattering coefficient.

albedo. These two parameters regulate the shape of the BRDF,  $k$  determining the amount of anisotropy in the BRDF and  $w_1$  the amount of forward with respect to backward scattering.  $r_0$  is the overall pixel reflectance. These results imply that if data are noisy and these results represent a realistic assessment, it may be difficult to interpret spatial maps of modified RPV model parameters. Rather, the derived quantities reflectance and albedo should be plotted to investigate the surface.

Figure 3 shows the red and near-infrared band noise amplification factors as a function of latitude for a 16-day period beginning on the first day of the year for the four land cover types used (solid and dotted curves). Also given is the result for the RossThick-LiSparse Ambrals model (dashed curve). These plots again demonstrate that the overall noise sensitivity behaviour of the two models is very similar. Where one model displays an increased sensitivity to noise, the other one does, too. This demonstrates that the reason for the noise sensitivity is not primarily a deficit of the model, but the lack of information inherent in the respective distribution of angular samples, affecting both models in the same way. These noise amplification factors are therefore valid specifically for MODIS-MISR sampling and will be different for other types of sampling.

Figure 2 shows the sensitivity of the three modified RPV model parameters. The

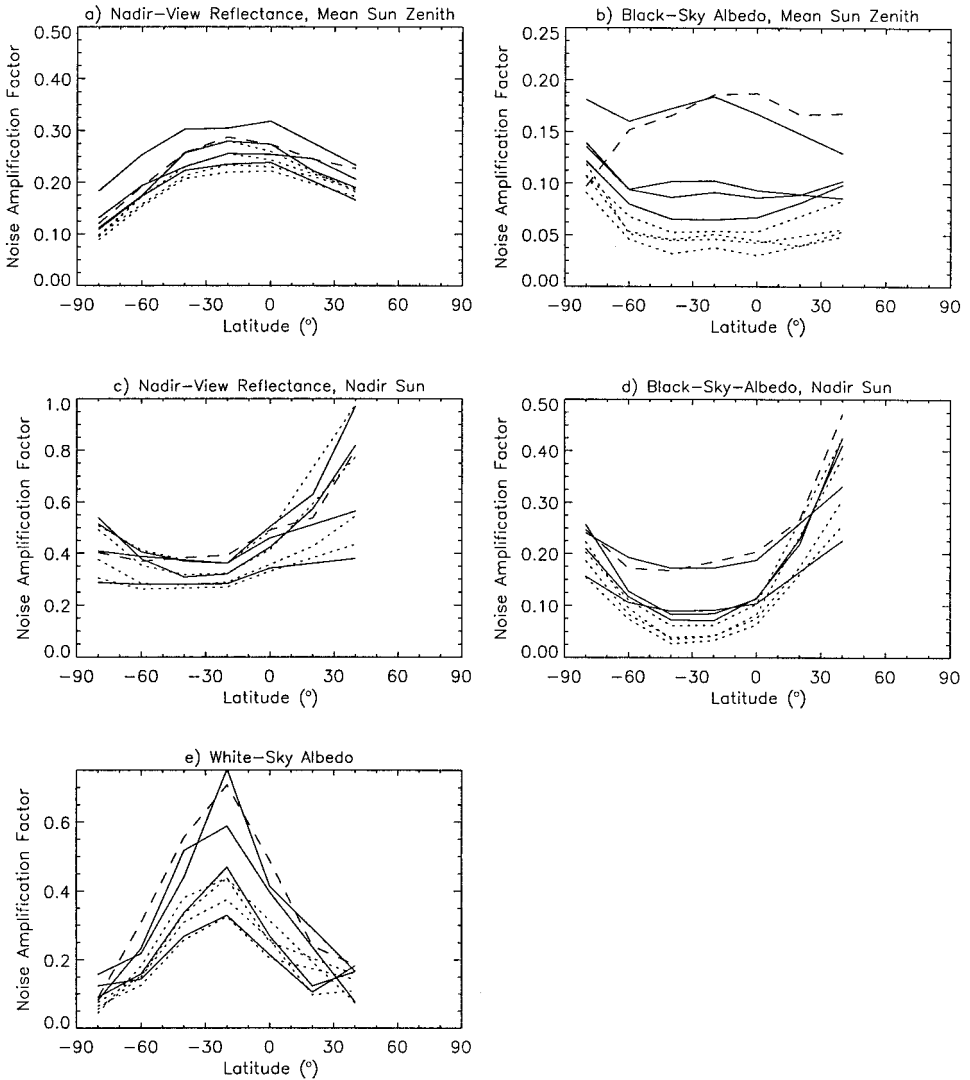


Figure 3. Noise sensitivity of the modified RPV BRDF model. Inferred equivalent weights of determination (noise amplification factors) are shown as a function of latitude for the first 16-day period of the year and for the red (solid curves) and near-infrared band (dotted curves) of four different land cover types. Also shown are the weights of determination for the Ambrals RossThick-LiSparse BRDF model (dashed curves). The terms ‘mean sun zenith’ and ‘nadir sun’ refer to evaluations at the mean sun zenith angle at which the observations were made at each respective latitude and for the case of a solar zenith angle extrapolated to nadir.

second and third parameters, the two describing BRDF shape, are extremely susceptible to noise, with noise amplification factors of more than 20 in the worst case, and larger than unity in most cases. This does not translate to noisy retrievals of reflectance and albedo, but will make it very difficult to use and interpret them directly. The cause of this sensitivity is probably internal redundancy in the way these parameters affect overall BRDF shape, perhaps caused by the hotspot term

in the model which allows trade-offs between parameters under limited angular sampling. However, since the model retrieves albedo and reflectance very well, this does not constitute a major problem in terms of physical quantities to be retrieved.

## 5. Conclusions

Due to the importance of the BRDF effect for state-of-the-art processing of remotely sensed data, several extensive efforts are currently being undertaken to produce global BRDF datasets for use by the earth science and remote sensing communities. For example, the Polarization and Directionality of the Earth's Radiation (POLDER) instrument will provide an extensive BRDF dataset derived from the Roujean BRDF model (Roujean *et al.* 1992) for the period the sensor on ADEOS-1 was in operation at 7 km resolution. MODIS will provide global 16-daily BRDF datasets with 1 km spatial resolution using the Ambrals BRDF model (Wanner *et al.* 1995). MISR will deliver global 9-daily BRDF datasets derived from a modified RPV model (Rahman *et al.* 1993). These BRDF data products will be used for standardizing the data acquired to a common geometry for uniform evaluation and mosaicking, for calculating land surface albedo as weighted integrals of the BRDF, for performing coupling between atmospheric radiative transfer and land surface reflectance, and for inferring land surface properties, mostly of vegetation, that influence the BRDF. Regional studies using Advanced Very High Resolution Radiometer Data (Li *et al.* 1996, d'Entremont *et al.* 1999, Hu *et al.* 1999b, Lewis and de Lope 1997) are successful precursors to the operational products.

As part of an ongoing effort to determine the magnitude of effects with a potentially negative impact on the quality of the MODIS BRDF/albedo product and similar algorithms we investigated the sensitivity of reflectance and albedo retrievals using the Ambrals RossThick-LiSparse BRDF model. The mathematical properties of linear kernel-driven BRDF models allow the analytical calculation of so-called weights of determination (in effect, noise inflation factors) that characterize the variance of physical properties resulting from the inversion as a function of the variance of the observations made. Such variations may result, for example, from fluctuations in surface properties, misregistration, atmospheric correction errors and other similar effects.

Results show that BRDF and albedo retrievals are stable with respect to random noise variations in the observed reflectances under the angular sampling schemes investigated, noise amplification factors generally being less than unity. This holds both for retrieval of BRDF and albedo at the mean sun angle of observation and for extrapolation of the retrieval to a nadir sun zenith angle. Where the Ambrals BRDF model shows increased susceptibility to noise, the modified RPV model, which was studied for comparison, does as well, indicating that the source of the problem lies in the geometric distribution of angular samples available, not with the model.

Generally, the modified RPV model is as stable with respect to noisy observations and MODIS and MISR sampling as the Ambrals BRDF model. The respective model parameters themselves are more noisy than the derived quantities BRDF and albedo for both models, but much more so for two of the three parameters of the modified RPV model. In terms of using different instruments for sampling, a combination of MODIS and MISR leads to excellent retrievals in terms of noise sensitivity. Using MISR only is also feasible. Using MODIS alone increases the noise sensitivity owing to the less favourable angular sampling properties of this instrument.

Taken together with other studies previously published (Hu *et al.* 1997, Lucht 1998; Hu *et al.* 1999a), this noise sensitivity study completes a series of investigations aimed at understanding the various influences on BRDF/albedo retrieval accuracies produced by the sparse angular sampling usually provided by remote sensing from space. As the operational phase of the EOS sensors approaches, we will be increasingly capable of quantifying the anisotropic properties of land surface reflectance globally, adding greatly to our understanding of the radiometric properties of the coupled biosphere–atmosphere system. These in turn may then be the basis for the modelling of the many non-radiometric aspects of the global Earth system we are interested in. Albedo, especially, stands out as a product of particular importance if it can be accurately retrieved using satellite observations.

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