CHAPTER 6

Geometry of Aerial Photographs*

6.1 SCALE EXPRESSIONS

Map or photo scale can be stated in one of two expressions as applied to aerial mapping: representative fraction or engineers' scale.

6.1.1 Representative Fraction

Representative fraction is expressed as a ratio in the form of 1:2400, where one unit on the photo or map represents 2400 similar units on the ground. For example:

1 in. on the map/photo = 2400 in. on the ground
1 ft on the map/photo = 2400 ft on the ground
1 m on the map/photo = 2400 m on the ground

6.1.2 Engineers' Scale

Engineers' scale is expressed as a ratio in the form of 1 in. = 200 ft, where one unit on the photo or map represents a number of different units on the ground.

6.1.3 Scale Conversion

Both of the examples of scale cited above mean the same thing. In order to convert from a representative fraction of 1:2400, assume that both are in inches. Then 2400 in. divided by 12 in. equals 200 ft, so the resultant engineers' scale is 1 in. = 200 ft. On the photo or map, 1 in. is equal to 200 ft on the ground.

Conversely, to convert from an engineers' scale of 1 in. = 200 ft, multiply 200 ft by 12 in. This simple arithmetic exercise equates 1 in. on the map or photograph to 2400 in. on the ground. The resultant representative fraction would be 1:2400.

* The geometry discussed in this chapter is reduced to several simple formulae that can be easily utilized in planning aerial photographic missions.

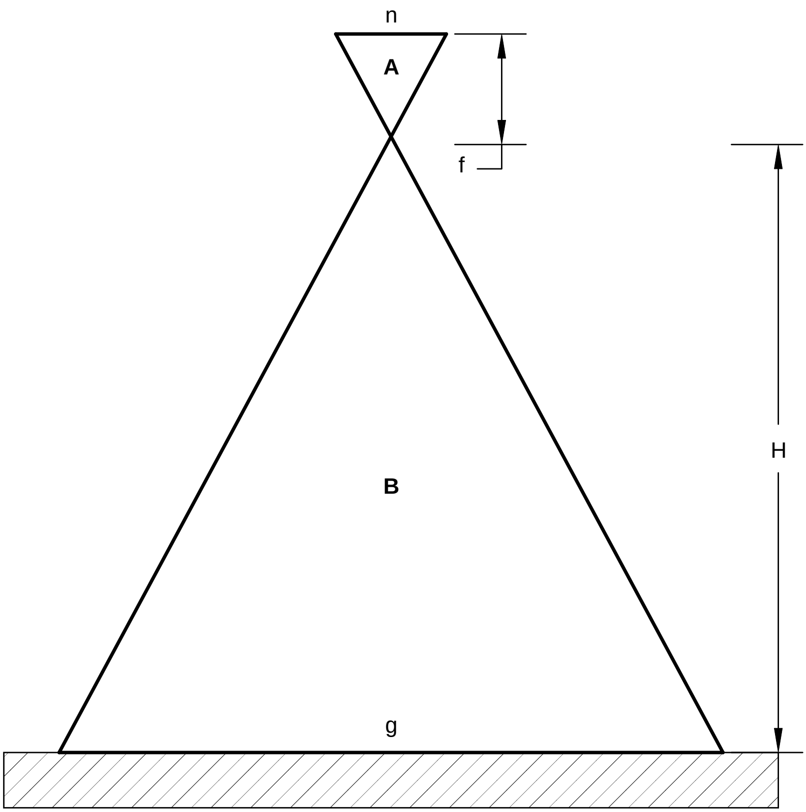


Figure 6.1 Geometry of negative scale.

6.2 GEOMETRY OF PHOTO SCALE

Figure 6.1 identifies similar triangles **A** and **B**. Analogous parts of similar triangles are proportional. Therefore, **n** (negative width) is proportional to **g** (ground distance covered by exposure) in the same magnitude as **f** (focal length) is to **H** (flight height above mean ground level).

6.2.1 Derivation of Photo Scale

The derivation of photo scale (s_p) with Equation 6.1 is a ratio which serves the purpose of determining negative scale based on negative width related to ground distance covered by the exposure frame.

$$s_p = \frac{n}{g} : \frac{1 \text{ in.}}{x \text{ ft}} \tag{6.1}$$

By factoring this ratio, the engineers' scale is 1 in. = x ft.

6.2.2 Controlling Photo Scale

Negative scale can be controlled by considering the specific mission characteristics. Negative scale is computed with the aid of Equation 6.2. This defines the relationship between focal length and flight height above mean ground level.

$$s_p = \frac{f}{H} : \frac{1 \text{ in.}}{x \text{ ft}} \tag{6.2}$$

By factoring the ratio, the photo scale is 1 in. = x ft.

6.2.2.1 Engineers' Scale

When using a 6-in. focal length camera flying at a height of 1200 ft above mean ground elevation, the engineers' scale of the photograph is:

$$s_p = \frac{f}{H} : \frac{6 \text{ in.}}{1200 \text{ ft}} = \frac{1 \text{ in.}}{200 \text{ ft}}$$

which translates to an engineers' scale of 1 in. = 200 ft.

6.2.2.2 Representative Fraction

This same situation can be utilized to calculate representative fraction, keeping in mind that a 6-in. focal length is equal to 0.5 ft:

$$s_p = \frac{f}{H} = \frac{0.5 \text{ ft}}{1200 \text{ ft}} = \frac{1 \text{ ft}}{2400 \text{ ft}}$$

which is analogous to a representative fraction of 1:2400.

6.2.3 Scale Formula

A simplified formula for determining photo scale, derived from the photo scale ratio, is presented in Equation 6.3.

$$s_p = H/f \tag{6.3}$$

where:

s_p = photo scale denominator (feet)
H = flight height above mean ground level (feet)
f = focal length of camera (inches)

Figure 6.2 Large-scale aerial photograph. (Courtesy of Surdex Corporation, Chesterfield, MO.)

6.2.4 Flight Height

For photos exposed with most precision aerial-mapping cameras, the calibrated focal length is noted in the margin of the exposure. A derivation of Equation 6.3 is Equation 6.4 for calculating flight height once the photo scale is selected.

$$H = s_p * f \tag{6.4}$$

6.2.5 Relative Photo Scales

There can be some confusion when thinking about relative photo scales. Just remember, large scale means that image detail is relatively large, and small scale means that image detail is relatively small. The discernable village that is visible on the photograph in Figure 6.2 appears on a large-scale aerial photo. Compare this with the photograph in Figure 6.3, which contains the same village in a small-scale aerial photo.

Figure 6.3 Small-scale aerial photograph. (Courtesy of Surdex Corporation, Chesterfield, MO.)

6.3 PHOTO OVERLAP

Aerial photo projects for all mapping and most image analyses require that a series of exposures be made along each of the multiple flight lines. To guarantee stereoscopic coverage throughout the site, the photographs must overlap in two directions: in the line of flight and between adjacent flights.

6.3.1 Endlap

Endlap, also known as forward overlap, is the common image area on consecutive photographs along a flight strip. This overlapping portion of two successive aerial photos, which creates the three-dimensional effect necessary for mapping, is known as a stereomodel or more commonly as a "model." Figure 6.4 shows the endlap area on a single pair of consecutive photos in a flight line.

Practically all projects require more than a single pair of photographs. Usually, the aircraft follows a predetermined flight line as the camera exposes successive overlapping images.

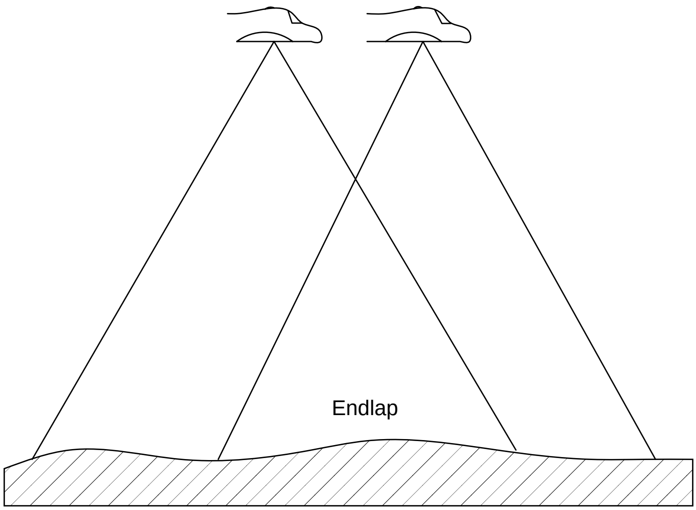


Figure 6.4 Endlap on two consecutive photos in a flight line.

Normally, endlap ranges between 55 and 65% of the length of a photo, with a nominal average of 60% for most mapping projects. Endlap gain, the distance between the centers of consecutive photographs along a flight path, can be calculated by using Equation 6.5.

$$g_{end} = s_p * w * \left[\left(100 - o_{end}\right) / 100\right] \quad (6.5)$$

where:

g_{end} = distance between exposure stations (feet)
s_p = photo scale denominator (feet)
o_{end} = endlap (percent)
w = width of exposure frame (inches)

When employing a precision aerial mapping camera with a 9 × 9 in. exposure format and a normal endlap of 60%, the formula is simpler. In this situation, two of the variables then become constants:

w = 9 in.
o_{end} = 60%

Then, the expression w*[(100% – o_{end})/100] becomes a constant equal to 3.6, and Equation 6.5 may be supplanted by Equation 6.6.

$$q_{end} = s_p * 3.6 \quad (6.6)$$

When utilizing a camera other than a 9 × 9 in. format and/or an endlap other than 60%, Equation 6.5 must be employed.

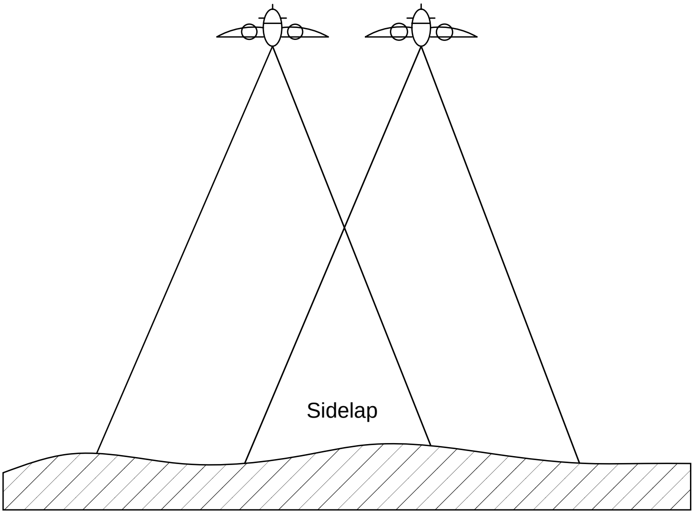


Figure 6.5 Sidelap between two adjacent flight lines.

Figure 6.6 Sidelap on three adjacent flight lines.

6.3.2 Sidelap

Sidelap, sometimes called side overlap, encompasses the overlapping areas of photographs between adjacent flight lines. It is designed so that there are no gaps in the three-dimensional coverage of a multiline project. Figure 6.5 shows the relative head-on position of the aircraft in adjacent flight lines and the resultant area of exposure coverage.

Usually, sidelap ranges between 20 and 40% of the width of a photo, with a nominal average of 30%. Figure 6.6 portrays the sidelap pattern in a project requiring three flight lines.

Sidelap gain, the distance between the centers of adjacent flight lines, can be calculated by using Equation 6.7.

$$q_{side} = s_p * w * \left[\left(100 - o_{side}\right)/100\right] \tag{6.7}$$

where:

g_{side} = distance between flight line centers (feet)
s_p = photo scale denominator (feet)
o_{side} = sidelap (percent)
w = width of exposure frame (inches)

When employing a precision aerial mapping camera with a 9 × 9 in. exposure format and a normal sidelap of 30%, the formula is simpler. In this situation, two of the variables then become constants:

w = 9 in.
o_{side} = 30%

Then, the expression w*[(100% – o_{side})/100] becomes a constant equal to 6.3, and Equation 6.7 may be supplanted by Equation 6.8.

$$q_{side} = s_p * 6.3 \tag{6.8}$$

When utilizing a camera other than a 9 × 9 in. format and/or a sidelap other than 30%, Equation 6.7 must be employed.

6.4 STEREOMODEL

From the foregoing discussion of overlap, it is evident that consecutive photos in a flight strip overlap. When focusing each eye on a particular image feature that was viewed by the camera from two different aspects, the mind of the observer is convinced that it is seeing a lone object with three dimensions. Put simply, the three-dimensional effect is an optical illusion. This phenomenon of observing a feature from different positions is known as the parallax effect. Although used to describe other facets of photogrammetry, parallax is defined as a change in the position of the observer. This situation allows a viewer, when using appropriate stereoscopic instruments, to observe a pair of two-dimensional photos and see a single three-dimensional image.

Photogrammetrists envision a model as the "neat" area that a single stereopair contributes to the total project. This allows for the endlap and sidelap with surrounding photos. A mapping model is shown as the crosshatched area in Figure 6.7.

Table 6.1 is a tabulation relating photo scale to flight height using a camera with a 6-in. focal length. For a given photograph, several parameters can be found:

- Flight height (above mean ground level)
- Photo center interval
- Flight line spacing
- Acres per model (neat area)

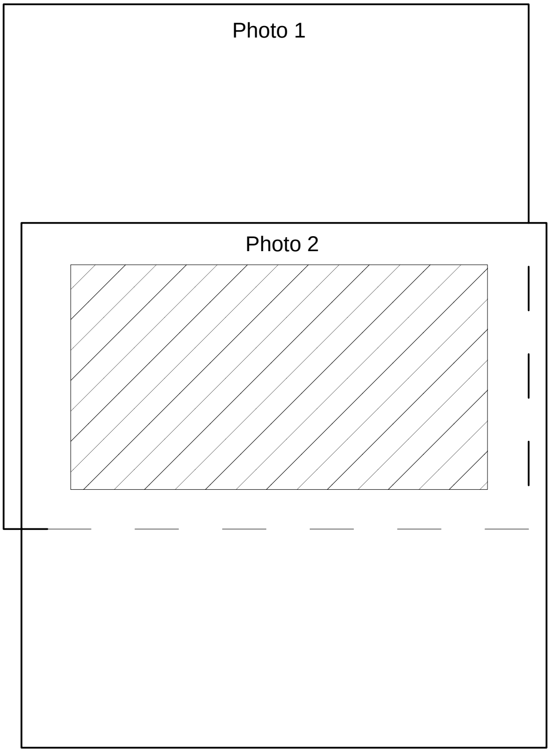


Figure 6.7 Neat area of a stereomodel.

It must be realized that the scale of individual photographs in a project is not a constant. Due to undulations in the aircraft flight and terrain relief, the distance between the camera and the ground differs from one exposure to another. Therefore, photo scale must be considered as an average scale for the total project.

6.5 RELIEF DISPLACEMENT

The surface of the earth is not smooth and flat. As a consequence, there is a natural phenomenon that disrupts true orthogonality of photo image features. In this respect, an orthogonal image is one in which the displacement has been removed, and all of the image features lie in their true horizontal relationship.

6.5.1 Causes of Displacement

Camera tilt, earth curvature, and terrain relief all contribute to shifting photo image features away from true geographic location. Camera tilt is greatly reduced or perhaps eliminated by gyroscopically-controlled cameras.

Table 6.1 Tabulation of Photo Scales with the Resultant Flight Height (Above Mean Ground), Endlap Gain, Sidelap Gain, and Acreage Per Neat Model

Scale 1 in. = *x* in.	Flight Height	Gain/Photo	Gain/Line	Acres/Model
167	1,000	601	1,052	14
200	1,200	720	1,260	21
250	1,500	900	1,575	32
300	1,800	1,080	1,890	47
350	2,100	1,260	2,205	64
400	2,400	1,440	2,520	83
450	2,700	1,620	2,835	105
500	3,000	1,800	3,150	130
550	3,300	1,980	3,465	158
600	3,600	2,160	3,780	187
650	3,900	2,340	4,095	220
700	4,200	2,520	4,415	255
750	4,500	2,700	4,725	293
800	4,800	2,880	5,040	333
850	5,100	3,060	5,355	376
900	5,400	3,240	5,670	422
1,000	6,000	3,600	6,300	521
1,250	7,500	4,500	7,875	813
1,320	7,920	4,753	8,316	907
1,500	9,000	5,400	9,450	1,171
1,667	10,000	6,000	10,500	1,446
2,000	12,000	7,200	12,600	2,983
2,500	15,000	9,000	15,750	3,245
3,000	18,000	10,800	18,900	4,685

Earth curvature is of little consequence on large-scale photography. The relatively small amount of lateral distance covered by the exposure frame introduces only a minimal amount of curvature, if any.

Topographic relief can have a great effect on displacing image features. The amount of image displacement increases on high-degree slopes. Feature displacement also increases radially away from the photo center.

6.5.2 Effects of Displacement

An aerial photograph is a three-dimensional scene transferred onto a two-dimensional plane. Hence, the photographic process literally squashes a three-dimensional feature onto a plane that lacks a vertical dimension, and image features above or below mean ground level are displaced from their true horizontal location. Figure 6.8 illustrates this phenomenon. Assuming that the stack rises straight into the air from the ground, both the top and the base possess the same horizontal (XY) placement. This diagram belies that fact, because the base and the top are in displaced positions (labeled “d” in Figure 6.8) on the negatives. This separation will not be of the same magnitude on successive photos.

Figure 6.9 illustrates the radial displacement of an object in an aerial photograph.

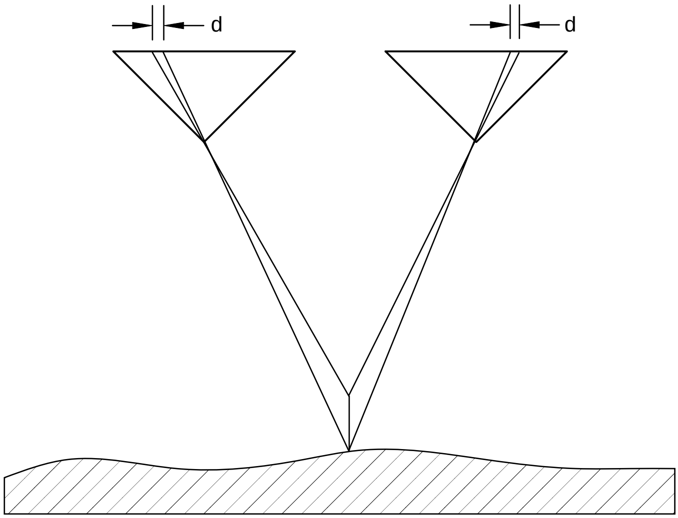


Figure 6.8 Image displacement.

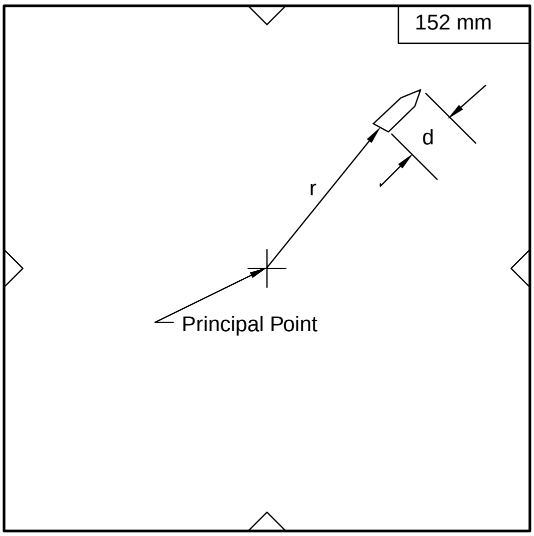


Figure 6.9 Radial displacement of an image feature.

Just as images of fast-rising features are displaced, so are the changes in ground elevations, though not as visibly apparent in the photographs. Figure 6.10 illustrates relief displacement on a straight utility clearing that crosses rolling hills. The clearing is identified as the wavy open strip running diagonally through the woods on the left side of the photo. Even though the indicated utility clearing follows a straight course, relief displacement due to terrain undulations causes this feature to waver.

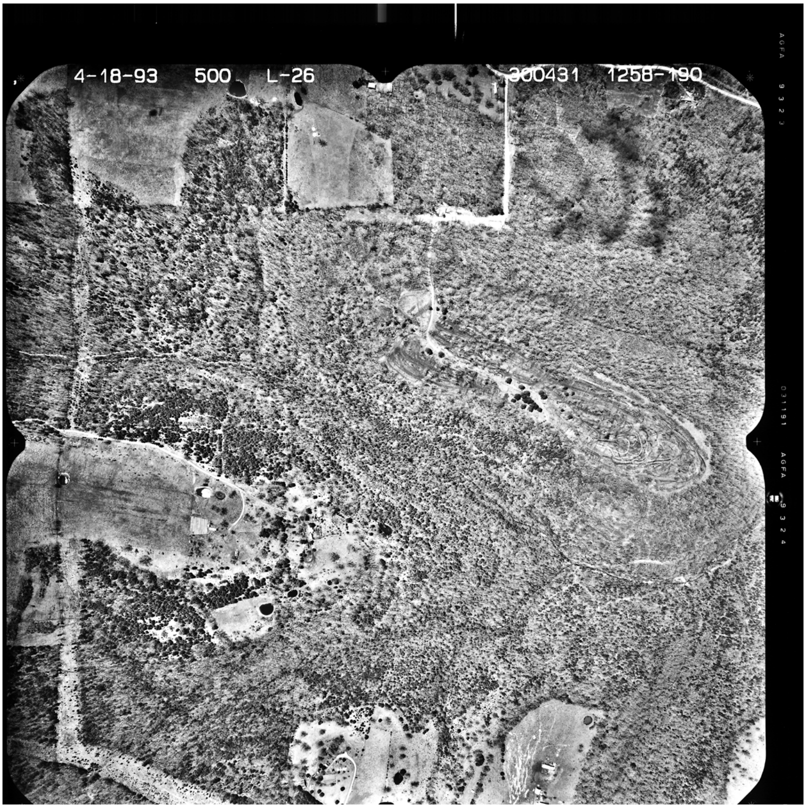


Figure 6.10 Image displacement on a utility clearing. (Courtesy of Surdex Corporation, Chesterfield, MO.)

Presuming this to be true, it follows that if several scale sets are calculated from an individual photograph, each may vary from the others. So, the more diverse the terrain character is, the more the scale variance.

6.5.3 Distortion vs. Displacement

Often, the term distortion is considered to be synonymous with displacement.

Distortion implies aberration. It is caused by discrepancies in the photographic, processing, and reproduction systems. This condition is not correctable in the compilation of a stereomodel.

Displacement is a normal inherent condition. Since mapping instruments work with a three-dimensional spatial image formed by a pair of overlapping two-dimensional photos, predictable displacement can be compensated for in the mapping process. Rather than being a fault in the image structure, displacement is the means by which it is possible to extract spatial information from photographs.

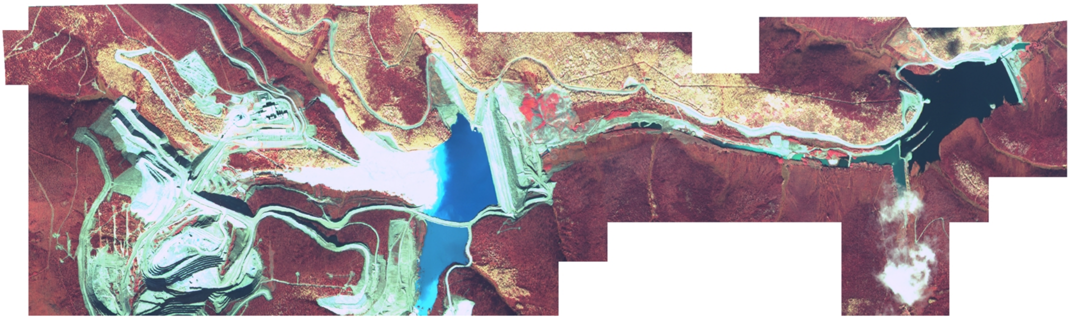
6.6 MEASURING OBJECT HEIGHT

Relief displacement allows the measurement of image object heights, either from a single photo or from a stereopair. Although photo interpreters in the past put these procedures to good use by manual methods, contemporary mappers are not directly concerned with the implementation of this approach because mapping instruments rely upon analytical solutions employing higher mathematics to achieve greater accuracy in processing differential parallax* comparator readings to create the same solution.

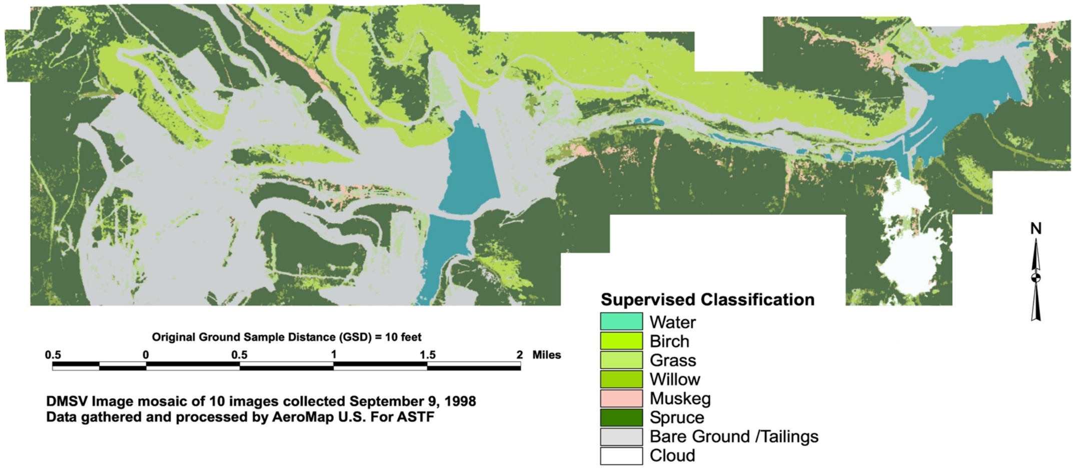
Differential parallax is an important concept in photogrammetric mapping. It allows the coordination of map features from images. Essentially, softcopy mappers and digital stereoplotters rely upon differential parallax to accomplish digital data collection.

* For a basic study of differential parallax refer to Chapter 6 in *Aerial Mapping: Methods and Applications,* Lewis Publishers, Boca Raton, FL, 1995.

Color Figure 1



Color Figure 2



Color Figure 3

Color Figure 1. Natural color imagery. This imagery is part of an Alaska Science and Technology Foundation grant, "Remote Sensing Technology for Mining Applications". John Ellis, of AeroMap, was the Project Manager. Courtesy of AeroMap U.S., Anchorage, AK, with permission.

Color Figure 2. False color imagery. This imagery is part of an Alaska Science and Technology Foundation grant, "Remote Sensing Technology for Mining Applications". John Ellis, of AeroMap, was the Project Manager. Courtesy of AeroMap U.S., Anchorage, AK, with permission.

Color Figure 3. A thematic map created by supervised classification procedures. This imagery is part of an Alaska Science and Technology Foundation grant, "Remote Sensing Technology for Mining Applications". John Ellis, of AeroMap, was the Project Manager. Courtesy of AeroMap U.S., Anchorage, AK, with permission.